Non-perfect M/M/R Machine repair problem with spares and two modes of failure

D.C.Sharma

Abstract-The paper investigates the machine repair problem consisting of M operating machines with S spare machine and R servers where machines have two failure modes and server are subjected to breakdown under steady state conditions. The two failure modes have equal probability of repair. Spares are considered to be cold standby or warm standby or hot standby. Failure and service time of machines and breakdown and repair time of server are assumed to follow a negative exponential distribution. Each server is subject to breakdown even if no failed machine is in the system.

Key words: Spares, cold standby, warm standby, hot standby

Introduction

In this paper we consider a group of identical machines (operating and spare machines) with two modes of failure and are maintained by one or more servers which are subjected to breakdown the repair facilities. The two failure modes of the machines have equal probability of repair. The spares are considered to be either cold standby, warm standby or hot standby. The system is considered to be closed if severs fail.

In a single mode of failure analytic steady state solution of M/M/R machine repair problem with cold standby and warm standby were first developed by Toft and Boothroyed1. For multiple modes of failure Benson and Cox2 proposed no spare M/M/1 machine repair problem without providing an analytic solution. The machine repair priority with exponentially distributed failure time and arbitrary distribution repair time was investigated by Jaiswal and Thiruvegandam3. Similar models with exponential failure and exponential repair time were studied by Elsayed4. Gaver and Lehoczky5 used diffusion approximation technique to study a repairman where failure may require two types of repair under repair time distribution with specific mean and covariance. The machine repair priority model under the assumption that the priority machines have pre-emptive priority over the ordinary ones were studied by Posafalvi and Sztrik6. Wang7 gave profit analysis of M/M/R machine repair problem with spares and server breakdowns. Wang and Wu8 studied the cost analysis of the M/M/R machine repair problem with spare and two modes of failure. Sharma and Sharma9 developed a model for M/M/R machine repair problem with spares and three modes of failure. MJ Armstrong10 studied age repair policies for the machine repair problem. The reliability analysis of balking and reneging in repairable system with warm standbys were studied by Kee and Wang11. SR Chakravrty12 gave analysis of a machine repair problem with unreliable server and phase type repair and services. N- Policy for machine repair system with spares and reneging was studied by M. Jain13. SP Chen14 gave a programing mathematical approach to the machine interference problem with fuzzy parameters. Reliability and sensitivity analysis of a repairable system with warm standby and R unreliable service stations were studied by Wang, Kee and Lee15. Again by Wang and Kee16 studied vacation policies for machine repair problem with two type spares. Machine repair problem in production system with spares and server vacations were studied by JC Ke and SL Lee17.

In this paper the problem is of interest to those systems where servers may allow to fail. This model is more realistic then that of Toft and Boothroyed1, Benson and Cox2, problem with spares (either cold standby, warm standby or hot standby) are divided for two failure modes. The results of this paper may be beneficial to those where the server is repairable and replacement is very costly.

Model and Equations

$N = M + S$ identical machines and R servers that are subjected to breakdown have been considered in this model, in which M operating machine and S are spare machines. Each operating or spare machine has two independent failure modes (mode1 and mode2). Operating machine as well as spares are subjected to breakdown with failure mode1, mode2 having independent exponential failure distributions with parameters $\lambda 1$ and $\lambda 2$ (operating machines) and $\lambda 1'$ and $\lambda 2'$ (for spares) respectively, where $0 \leq \lambda 1' \leq \lambda 1$ and $0 \leq \lambda 2' \leq \lambda 2$. The failed machines are repaired by the repairman on the FCFS basis. Suppose that both modes are equally likely to be repaired when several machines are waiting for repair. The service time for repair at this facilities are exponentially distributed with parameters μ1 and μ2 of failure of mode1 and mode2 respectively. Breakdown of the server takes place at any time with breakdown rate α. Whenever a server fails it is immediately repaired at a repair rate β.

M/M/1 Model

Let the state $s = 0$ and 1 represents the server in operation and failed respectively with states $\{\frac{i,j}{i} + j = 0, 1, 2, \ldots \ldots N\}$ where i denotes the number of failed machines of mode 1 and j denotes the number of failed machines of mode 2.

$P_0(i, j)$ =probability that there are i and j failed machines of mode 1 and mode 2 respectively in the system when the server is working.

$P_1(i, j)$ =probability that there are i and j failed machines of mode 1 and mode 2 respectively in the system when the server is broken down.

The steady equation for $P_s(i, j)$ $(s = 0, 1)$ for the non-perfect M/M/1 machines repair problem with spare and two modes of failure are given by

$$[M(\lambda_1 + \lambda_2) + S(\lambda'_1 + \lambda'_2) + \alpha]P_0(0,0) = \mu_1 P_0(1,0) + \mu_2 P_0(0,1) + \beta P_1(0,0) \quad (1)$$

$$[M(\lambda_1 + \lambda_2) + (S - i)(\lambda_1 + \lambda_2) + \alpha + \mu_1]P_0(i, 0) = \mu_1 P_0(i + 1,0) + \mu_2 P_0(i, \lambda)$$

$$+[M\lambda_1 + (S - i + \lambda)\lambda_1]P_0(i - 1, 0) + \beta P_1(i, 0); \quad 1 \le i \le S \quad (2)$$

$$[\alpha + \mu_1 + (N - i)(\lambda_1 + \lambda_2)]P_0(i, 0) = \mu_1 P_0(i + 1,0) + \mu_2 P_0(i, 1)$$

$$+(N - i + 1)\lambda_1 P_0(i - 1,0) + \beta P_1(i, 0);\ S < i < N \quad (3)$$

$$(\alpha + \mu_1)P_0(N, 0) = \lambda_1 P_0(N - 1,0) + \beta P_1(N, 0) \quad (4)$$

$$[M(\lambda_1 + \lambda_2) + (S - j)(\lambda'_1 + \lambda'_2) + \alpha + \mu_2]P_0(0, j) = \mu_1 P_0(0, j + 1) + \mu_2 P_0(1, j)$$

$$+[M\lambda_2 + (S - j + 1)\lambda_2]P_0(0, j - 1) + \beta P_1(0, j); \quad 1 \le i \le S \quad (5)$$

$\mu_1 P_0 (0, j+1) + \mu_2 P_0 (0, j) + (N-j+1) \lambda_2 P_0 (0, j-1) + \beta P_1 (0, j);$

$S < j < N$ (6)

$(\alpha + \mu_2)P_0 (0, N) = \lambda_2 P_0 (0, N-1) + \beta P_1 (0, N)$ (7)

$[\mu_1 + \mu_2 + M (\lambda_1 + \lambda_2) + (S-i-j) (\lambda'_1 + \lambda'_2) + \alpha] P_0 (i, 1)$

$= \mu_1 P_0 (i+1, j) + [M\lambda_1 + (S-i-j+1) \lambda'_1] P_0 (i-1, j)$

$+ \mu_2 P_0 (i, j+1) + [M\lambda_2 + (S-i-j+1) \lambda'_2] P_0 (i, j-1) + \beta P_1(i, j)$

$i, j \neq 0, 1 < i + j \le S$ (8)

$[\mu_1 + \mu_2 + (N-i-j) (\lambda_1 + \lambda_2) \alpha] P_0 (i, j)$

$= \mu_1 P_0 (i+1, j) + \mu_2 P_0 (i, j+1) + (N-i-j+1) [\lambda_1 P_0 (i-1, j) + \lambda_2 P_0 (i,j-1)] + \beta P_1 (i, j)$;$i, j \neq N,\ S < i + j \le N$ (9)

$[M\lambda_1 + M\lambda_2 + S (\lambda'_1 + \lambda'_2) + \beta] P_1 (0, 0) = \alpha P_0 (0, 0)$ (10)

$[M (\lambda_1 + \lambda_2) + (S-i) (\lambda'_1 - \lambda'_2) + \beta] P_1 (i, 0) = [M\lambda_1 + (S-i+1) \lambda'_1] P_1 (i-1, 0) + \alpha P_0 (i, 0);\ 1 \le i \le S$ (11)

$[\beta + (N-i) (\lambda_1 + \lambda_2)] P_1 (i, 0) = (N-i+1) \lambda_1 P_1 (i-1, 0) + \alpha P_0 (i, 0)$

$S < j < N$ (12)

$\beta P_1 (N, 0) = \alpha P_0 (N, 0) + \lambda_1 P_1 (N-1, 0)$ (13)

$[M (\lambda_1 + \lambda_2) + (S-j) (\lambda'_1 + \lambda'_2) + \beta] P_1 (0, j) = [M\lambda_2 + (S-j+1) \lambda'_2] P_1 (0, j-1) + \alpha P_0 (0, j);\ 1 \le j < S$ (14)

$[\beta + (N-j) (\lambda_1 + \lambda_2)] P_1 (0, j) = (N-j+1) \lambda_1 P_1 (0, j-1) + \alpha P_0 (0, j)$

$S < j < N$ (15)

$[M (\lambda_1 + \lambda_2) + (S-i-j) (\lambda'_1 + \lambda'_2) + \beta] P_1 (i, j)$

$= [M\lambda_1 + (S-i-j+1) \lambda_1] P_1 (i-1, j) + [M\lambda_2 + (S-i-j+1) \lambda_2] P_1 (i,j-1) + \alpha P_0 (i, j);\ i,j \le 0, 1 \le i+j \le S$ (16)

$[(N-i-j) (\lambda_1 + \lambda_2) + \beta] P_1 (i, j)$

$= (N-i-j+1) [\lambda_1 P_1 (i-1, j) + \lambda_2 P_1 (i,j-1)] + \alpha P_0 (i, j)$

$i, j \neq N,\ S < i + j \le N$ (17)

$\beta P_1 (i, j) = \alpha P_0 (i, j) (N-i+1) \lambda_1 P_1 (i-1, j) + (N-j+1) \lambda_2 P_1 (i, j-1)$ (18)

If $\lambda_1=0, \lambda_2=0$, S=0 and $\alpha = 0$ and $\beta=0$, we obtain the result $P_0 (i, j)$ for perfect M/M/1, no spare, single server model with two failure modes when $\lambda'_1 = 0, \lambda'_2 = 0$, S = 0 we reduce to the result $P_0 (i, j)$ for non-perfect model with two failure modes and with no spare.

For cold standby model ($\lambda'_1 = 0$, $\lambda'_2 = 0$) in two modes of failure with server breakdown (non-perfect model).

From (1) $[M \lambda_1 + \alpha] P_0 (0, 0) = \mu_1 P_0 (1, 0) + \beta P_1 (0, 0)$

From (10) $[M \lambda_1 + \beta] P_1 (0, 0) = \alpha P_0 (0, 0)$

$P_1 (0, 0) = \frac{\alpha}{M\lambda 1+\beta} P_0 (0, 0)$

$[M\lambda_1 + \alpha] P_0 (0, 0) = \mu_1 P_0 (1, 0) + \frac{\alpha\beta}{M\lambda 1+\beta} P_0 (0, 0)$

$\mu_1 P_0 (1, 0) = [(M\lambda_1 + \alpha) - \frac{\alpha\beta}{M\lambda 1+\beta}] P_0 (0, 0)$

$= \frac{(M\lambda 1)2 + M\lambda 1(\alpha+\beta)}{M\lambda 1 + \beta} P_0 (0, 0)$

$= \frac{[M\lambda 1 + (\alpha+\beta)]}{[M\lambda 1 + \beta]} M\lambda_1 P_0 (0, 0)$

$= [1 + \frac{\alpha}{M\lambda 1+\beta}] M\lambda_1 P_0 (0, 0)$

$= M\lambda_1 [1 + \frac{\alpha}{M\lambda 1+\beta}] P_0 (i, 0)$

$\mu_1 P_0 (i+1, 0) = [M\lambda_1 [1 + \frac{\alpha}{M\lambda 1+\beta}] + \mu_1] P_0 (i, 0)$

$- M\lambda_1 \{P_0 (i-1, 0) + \frac{\beta}{M\lambda 1+\beta} P_1 (i-1, 0)\}$

$P_0 (i+1, 0) = [1+ \frac{M\lambda 1}{\mu 1} \{1 + \frac{\alpha}{M\lambda 1+\beta}\}] P_0 (i, 0)$

$- \frac{M\lambda 1}{\mu 1} \{P_0 (i-1, 0) + \frac{\beta}{M\lambda 1+\beta} P_1 (i-1, 0)\}; 1 < i \le S$ (19)

Where $P_1 (i, 0) = \frac{M\lambda 1}{M\lambda 1+\beta} P_0 (i-1, 0) + \frac{\alpha}{M\lambda 1+\beta} P_0 (i, 0)$

and $P_1 (0, 0) = \frac{\alpha}{M\lambda 1+\beta} P_0 (0, 0)$

$P_0 (1, 0) = \frac{M\lambda 1}{\mu 1} \{1 + \frac{\alpha}{M\lambda 1+\beta}\} P_0 (0, 0)$ (20)

Similarly for $P_0 (0, j+1)$ we can write

$P_0 (0, j+1) = [1+ \frac{M\lambda 2}{\mu 2} \{1 + \frac{\alpha}{M\lambda 2 + \beta}\}] P_0 (0, j)$

$- \frac{M\lambda 2}{\mu 2} [P_0 (0, j-1) + \frac{\beta}{M\lambda 2 + \beta} P_1 (0, j-1)]; 1 \le j \le S$ (21)

Where $P_1 (0, j) = \frac{M\lambda 2}{M\lambda 2+\beta} P_1 (0, j-1) + \frac{\alpha}{M\lambda 2+\beta} P_0 (0, j)$

and $P_1 (0, 0) = \frac{\alpha}{M\lambda 2+\beta} P_0 (0, 0)$

$P_0 (0, 1) = \frac{M\lambda 2}{\mu 2} \{1 + \frac{\alpha}{M\lambda 2+\beta}\} P_0 (0, 0)$ (22)

$P_0 (i+1, 0) = \frac{[\alpha + \mu 1 + (N-i)\lambda 1]}{\mu 1} P_0 (i, 0)$

$-(N-i+1)\frac{\lambda 1}{\mu 1} P_0 (i-1, 0) - \frac{\beta}{\mu 1} P_1 (i, 0); S < i < N$ (23)

Where $P_1 (i, 0) = \frac{(N-i+1)\lambda 1}{(N-i)\lambda 1+\beta} P_1 (i-1, 0)$

$+ \frac{\alpha}{(N-i)\lambda 1+\beta} P_0 (1, 0)$ (24)

Similarly we get for

$P_0 (0, j+1) = (1 + \frac{[\alpha + (N-j)\lambda 2]}{\mu 2}) P_0 (0, j) - (N-j+1)\frac{\lambda 2}{\mu 2} P_0 (0, j-1) - \frac{\beta}{\mu 2} P_1 (0, j); S < j < N$

(25)

and $P_1 (0, j) = \frac{(N-j+1)\lambda 2}{(N-j)\lambda 2+\beta} P_1 (1, j-1)$

$+ \frac{\alpha}{(N-j)\lambda 2+\beta} P_0 (0, j); S < j < N$ (26)

Now from

(1), (8), (16) we get $P_0 (i+1, j+1), \quad 1 \le i + j \le S$

(1), (9), (17) we get $P_0 (i+1, j+1), \quad S \le i + j \le N$

M/M/R Model

Let the states, s (s = 0, 1….. , R) represents that s servers are broken down while the states $\{(i, j)/ i + j = 0, 1, 2……, N\}$ where i denotes the number of failed machines of mode 1 and j denotes the number of failed machines of mode 2.

$P_1 (i, j)$ = probability that there are i failed machines of mode 1 and j failed machines of mode 2 in the system when the servers are working.

$P_s (i, j)$ = probability that there are i failed machines of mode 1 and j failed machines of mode 2 respectively where s servers are broken.

s = 0, 1….., R-1, i+j = 0,1,2 …….N.

$P_R (i, j)$ = when R servers are broken.

Now the non-perfect M/M/R machine repair problem with spares, are given by

(i) S = 0

$[M (\lambda_1 + \lambda_2)+S (\lambda'_1+\lambda'_2) + R\alpha] P_0 (0, 0)]$

$= \mu_1 P_0 (1, 0) + \mu_2 P_0 (0, 1) + \beta P_1 (0, 0);$ (1)

$\min (i, R) \mu_1 + [M\lambda_1 + (S-i) \lambda'_1] + [M\lambda_2 + (S-i) \lambda'_2$

$+ R\alpha] P_0 (i, 0)$

$= \min (i+1, R) \mu_1 P_0 (i+1, 0) + \mu_2 P_0 (i, 1) + [M\lambda_1 + (S-i+1) \lambda'_1] P_0 (i-1, 0) + \beta P_1 (i, 0); 1 \le i \le S$ (2)

$[\min (i, R) \mu_1 + (N-i) (\lambda_1 + \lambda_2)] P_0 (i, 0) + R\alpha P_0 (i, 0)$

$= \min (i+1, R) \mu_1 P_0 (i+1, 0) + \mu_2 P_0 (i, 1) + (N-i+1) \lambda_1 P_0 (i-1, 0) + \beta P_1 (i, 0); S < i < N$ (3)

$(R\mu_1 + R\alpha) P_0 (N, 0) = \lambda_1 P_0 (N-1, 0) + \beta P_1 (N, 0)$ (4)

$[\min(i, R)\ \mu_2 + M(\lambda_1 + \lambda_2) + (S-j)(\lambda'_1 + \lambda'_2) + R\alpha]\ P_0(0, j)$

$= \min(j+1, R)\ \mu_2 P_0(0, j+1) + \mu_2 P_0(1, j) + [M\lambda_2 + (S-j+1)\lambda'_2]\ P_0(0, j-1) + \beta P_1(0, j);\ 1 \le j \le S$ (5)

$[\min(j, R)\ \mu_2 + (N-j)(\lambda_1 + \lambda_2) + R\alpha]\ P_0(0, j)$

$= \mu_1 P_0(i, j) + \min(j+1, R)\ \mu_2 P_0(0, j+1) + (N-j+1)\lambda_2 P_0(0, j-1)$

$+ \beta P_1(0, j);\ S < j < N$ (6)

$[R\mu_2 + R\alpha]\ P_0(0, N) = \lambda_2 P_0(0, N-1) + \beta P_1(0, N)$ (7)

(ii) $1 \le s \le R-1$

$[M(\lambda_1 + \lambda_2) + S(\lambda'_1 - \lambda'_2) + (R-s)\alpha + s\beta]\ P_s(0, 0)]$

$= (R-s+1)\alpha P_{s-1}(0, 0) + (S+1)\beta P_{s+1}(0, 0) + \mu_1 P_s(1, 0)$

$+ \mu_2 P_s(0, 1)$ (8)

$[\min(i, R-s)\ \mu_1 + [M(\lambda_1 + \lambda_2) + (S-i)(\lambda'_1 + \lambda'_2) + (R-s)\alpha + s\beta]$

$P_s(i, 0)$

$= \min(i+1, R-s)\ \mu_1 P_s(i+1, 0) + \mu_2 P_s(i, 1) + [M\lambda_1$

$+ (S-j+1)\lambda_1]\ P_s(i-1, 0) + (s+1)\beta P_{s+1}(i, 0); 1 \le j < R-(S+1)$ (9)

$[\min(i, R-s)\ \mu_1 + (N-i)(\lambda_1 + \lambda_2) + (R-s)\alpha + s\beta]\ P_s(i, 0)$

$= \min(i+1, R-s)\ \mu_1 P_s(i+1, 0) + \mu_2 P_s(i, 1) + (N-i+1)\lambda_1\ P_s(i-1, 0) + (R-s+1)\alpha P_{s+1}(i, 0) + (s+1)\beta P_{s+1}(j, 0)$ (10)

$[(R-s)\mu_1 + (R-s)\alpha + s\beta]\ P_s(N, 0) = \lambda_1 P_s(N-1, 0)$

$+ (R-s+1)\alpha P_{s-1}(N, 0) + (s+1)\beta P_{s+1}(N, 0$(11)

$[\min(j, R-s)\ \mu_2 + M(\lambda_1 + \lambda_2) + (S-j)(\lambda_1 + \lambda_2) + (R-s)\alpha + s\beta]\ P_s(0, j)$

$= \min(j+1, R-s)\ \mu_2 P_s(0, j+1) + \mu_1 P_s(1, j) + [M\lambda_2 + (S-j+1)\lambda'_2]\ P_s(0, j-1) + (s+1)\beta P_s(0, j);\ 1 \le j < R-(S+1$ (12)

$[\min(j, R-s)\ \mu_2 + (N-j)(\lambda_1 + \lambda_2) + (R-s)\alpha + s\beta]\ P_s(0, j)$

$= \min(j+1, R-s)\ \mu_2 P_s(0, j+1) + \mu_1 P_s(1, j) + (N-j+1)\lambda_2\ P_s(1, j-1) + (R-s+1)\alpha P_s(0, j) + (s+1)\beta P_{s+1}(0, j)$ (13)

$[(R-s)\mu_2 + (R-s)\alpha + s\beta]\ P_s(0, N) = \lambda_2 P_s(0, N-1) + (R-s+1)\alpha P_{s-1}(0, N) + (s+1)\beta P_{s+1}(0, N)$ (14)

$[\min(i, R-s)\ \mu_1 + \min(j, R-s)\ \mu_{2+}\ [M\lambda_1 + (S-i-j)\lambda'_1]$

$+ (M\lambda_2 + (s-1-j)\lambda'_2) + (R-s)\alpha + s\beta]\ P_s(i, j)$

$= \min(s+1, R-s)\mu_1 P_s(i+1, j) + [M\lambda_1 + (s-i-j+1)\lambda'_1]$

$P_s(i-1, j) + \min(j+1, R-s)\mu_2 P_s(i, j+1) + [M\lambda_2 + (s-i-j+1)\lambda'_2]\ P_s(i, i-1) + (R-s-1)\alpha P_{s-1}(i, j) + (s+1)\beta P_s(i, j);\ i, j \ne 0,\ 1 \le i+j \le R-s$ (15)

$[\min(i, R-s)\mu_1 + \min(i, R-s)\ \mu_2 + (N-i-j)(\lambda_1 + \lambda_2) + (R-s)\alpha + s\beta]\ P_s(i, j)$

$= \min(i+1, R-s)\mu_1 P_s(i+1, j) + \min(i+1, R-s)\mu_2\ P_s(i, j-1)$

$+ (N-i, j+1)[\lambda_1 P_s(i-1, j) + \lambda_2 P_s(i, j-1)$

$+ (R-s+1)\alpha P_{s+1}(i, j) + (s+1)\beta P_{s+1}(i, j) i, j \ne N\quad R-s \le i+j \le N$ (16)

(iii) s=R

$[M(\lambda_1 + \lambda_2) + S(\lambda'_1 + \lambda'_2) + R\beta]\ P_R(0,0) = P_{R-1}(0,0)$ (17)

$[M(\lambda_1 + \lambda_2) + (S-i)(\lambda'_1 + \lambda'_2) + R\beta]\ P_R(i,0)$

$= [M\lambda_1 + (S-i)\lambda_1)]P_R(i-1,0) + \alpha P_{R-1}(i,0);\ 1 \le j \le N-1$ (18)

$[M(\lambda_1 + \lambda_2) + (S-j)(\lambda'_1 + \lambda'_2) + R\beta]\ P_R(0,j)$

$= [M\lambda_2 + (S-j)\lambda'_2)]P_R(i-1,0) + \alpha P_{R-1}(0,1); 1 \le j \le N-1$ (19)

$[M(\lambda_1 + \lambda_2) + (S-i-j)(\lambda'_1 + \lambda'_2) + R\beta]\ P_R(i,j)$

$= [M\lambda_1 + (S-i-j+1)\lambda'_1)]P_R(i-1,j) + [M\lambda_2 + (S-i-j+1)\lambda'_2)]P_R(i,j-1) + \alpha P_{R-1}(i,j); i, j \ne 0, 1 \le i+j \le S$ (20)

$[(N-i-j)(\lambda_1 + \lambda_2)]\ P_R(i,j) = (N-i-j+1)\ [\lambda_1 P_R(i-1,j) + \lambda_2 P_R(i,j-1)] + \alpha P_{R-1}(i,j)\ ;\ i, j \ne N\quad S \le i+j < N$ (21)

$R\beta P_R(N,0) = \lambda_1 P_R(N-1,0) + \alpha P_{R-1}(N,0)$ (22)

$R\beta P_R(0,N) + \lambda_2 P_R(0,N-1) + \alpha P_{R-1}(0,N)$ (23)

The M/M/R model is solvable recursively for R = 1 but it is not possible to solve the model in general. We require a computer program for R > 1.

Conclusions

This model generalizes the perfect M/M/R machine repair model with spares and two modes of failures. Generalised solutions for the non-perfect M/M/R machine repair problem with two modes of failure is obtained for R = 1. Solution for R > 1, we require computer program.

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